

# MicroDrift with Bayesian Covertrees

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CAMLIS, 2021

# About Me

- ▶ Ph.D. in Algebraic Topology from JHU
- ▶ Very involved in the AI Village
- ▶ Formerly at Endgame / Elastic

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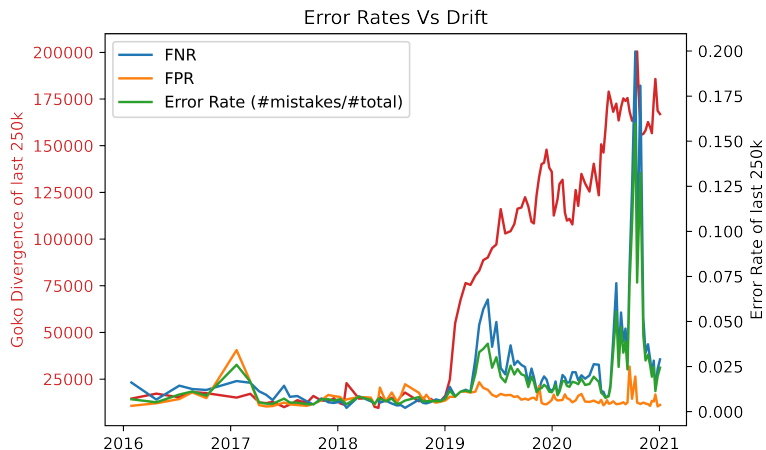
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# Problem Formulation



# Previous Work: Chronological Drift



Work done at Elastic, published at ICLR

# Problems With Previous Work

- ▶ Doesn't model the efficacy metrics well
- ▶ Not that actionable, just "Retrain when KL-Div exceeds X"
- ▶ There's way more detail than just a single metric in the method

# Objective of This Talk

Tell me where there's a problem in my dataset, not just that there's a problem.

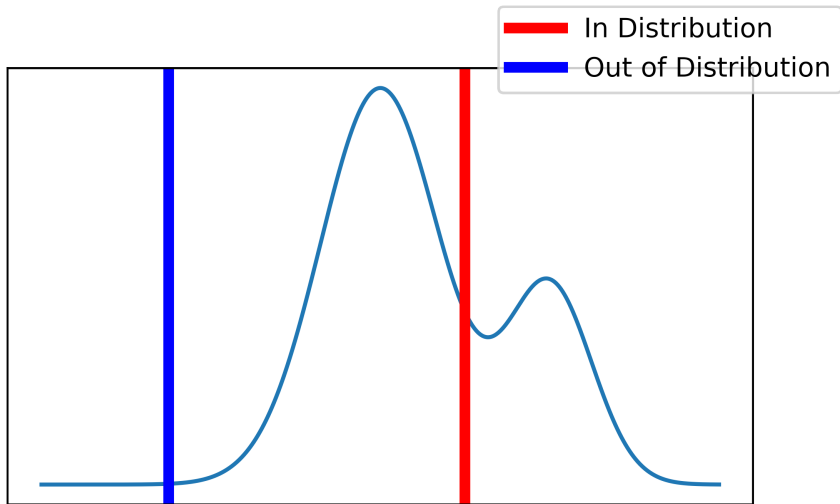
Where am I being attacked/bypassed?

Where is that new malware family?

Where is that new popular spam technique?



# Types of Bypass



# What I'm Actually Doing

- ▶ We have a dataset, and model.
- ▶ Queries stream in from anonymous users.
- ▶ One user has an in-distribution "bypass" they are repeating.
  - ▶ Building an attack with ZOO, or HopSkipJump.
  - ▶ Spamming their spam everywhere.
- ▶ The bad user's queries only account for a small percentage of total traffic.
- ▶ *We want to isolate that user's queries as best as possible.*

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## Definition

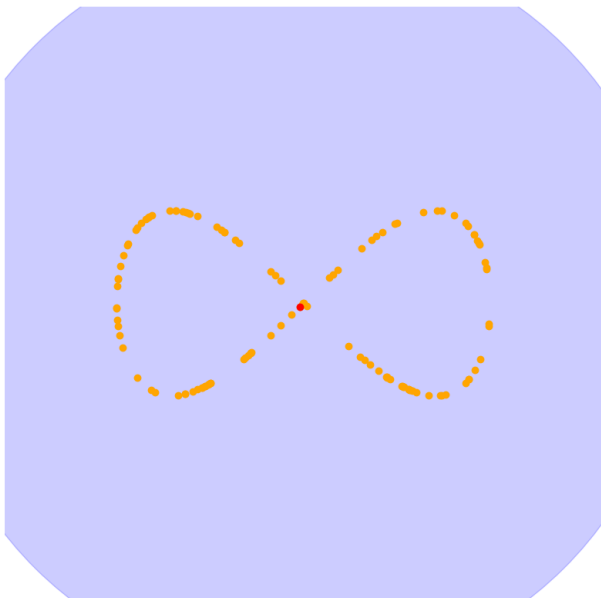
A *cover tree* over a dataset  $X = \{x_1, \dots, x_n\}$  is a filtration of a dataset into  $m$ -layers, with a scale base of  $S$

$$\{x_r\} = C_k \subset C_{k-1} \subset \dots \subset C_{k-m} = X,$$

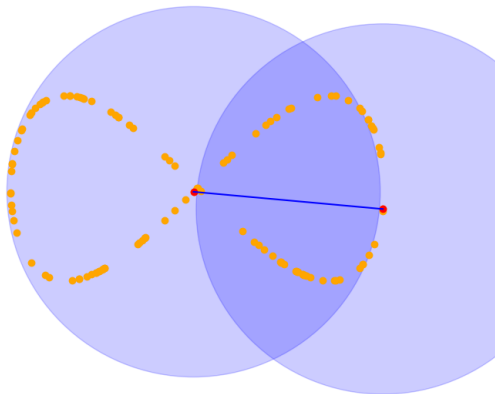
which satisfies the following properties:

1. *Covering Layer*: For each  $x_j \in X$  and  $i \in \{k, \dots, k - m\}$ , there exists  $p \in C_i$  such that  $d(x_j, p) < s^i$ .
2. *Covering Tree*: For each  $p \in C_{i-1}$  there exists  $q \in C_i$  such that  $d(p, q) < s^i$ .
3. *Separation*: For all  $p, q \in C_i$ ,  $d(p, q) > s^i$ .

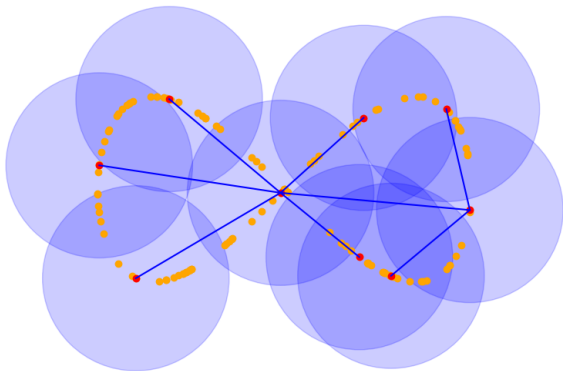
Lets's build one, Level 1



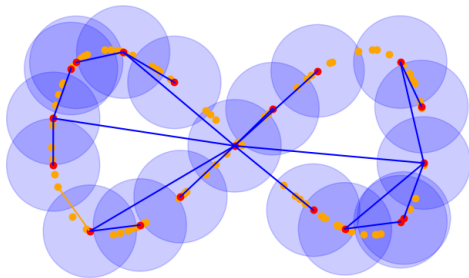
Lets's build one, Level 0



Lets's build one, Level -1

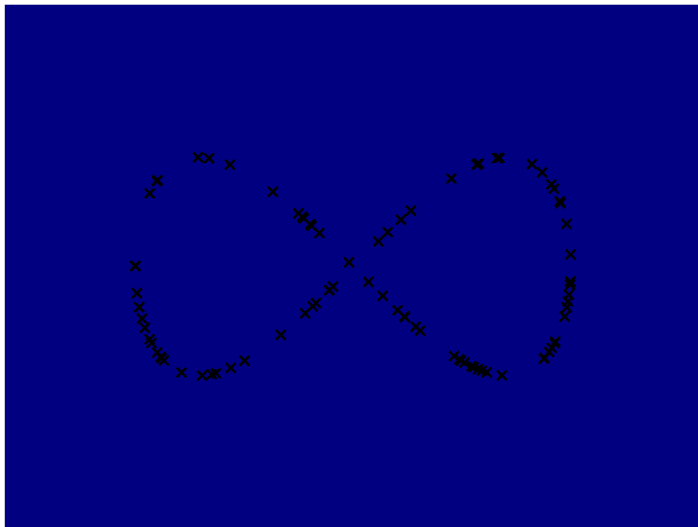


## Lets's build one, Level -2

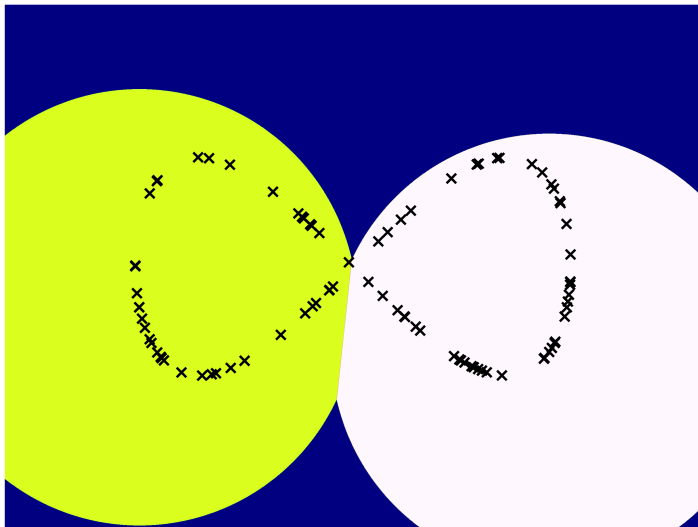




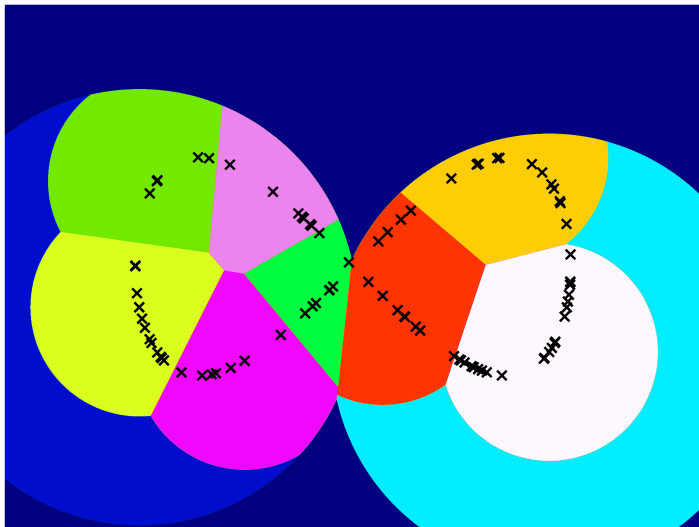
# How A Covertree Partitions Space, Level 1



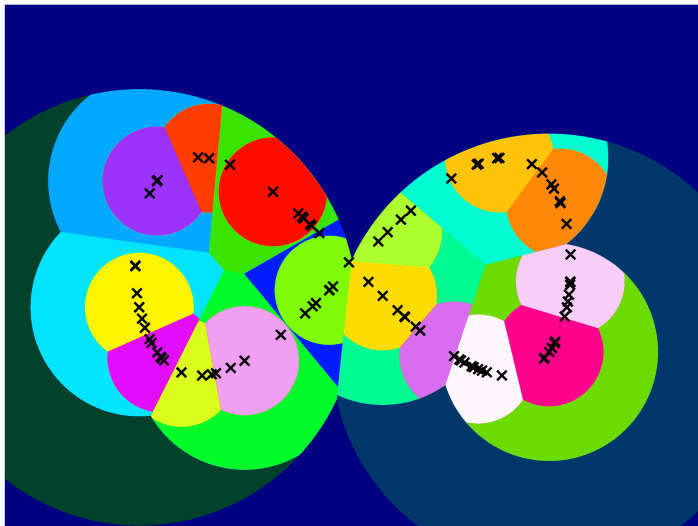
# How A Covertree Partitions Space, Level 0



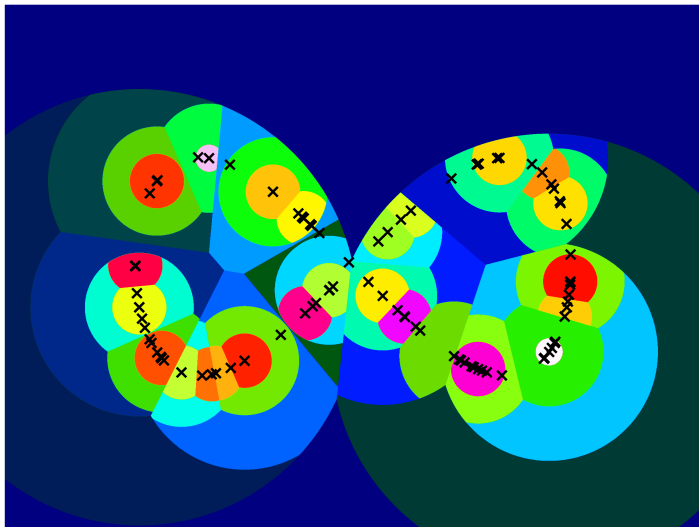
# How A Covertree Partitions Space, Level -1



## How A Covertree Partitions Space, Level -2



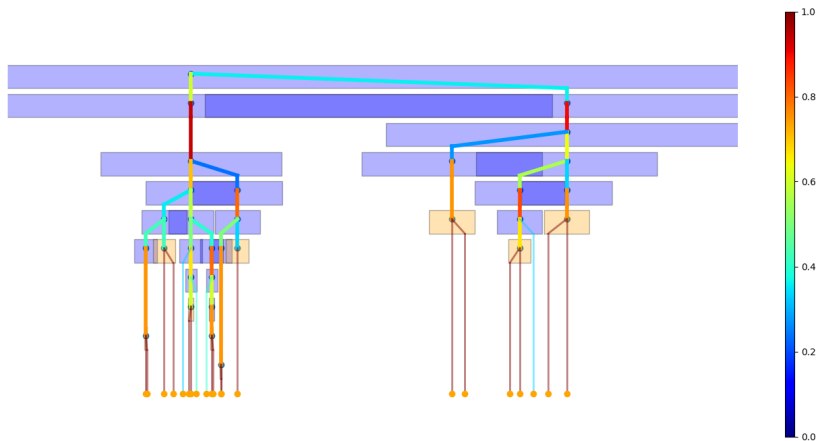
# How A Covertree Partitions Space, Level -3



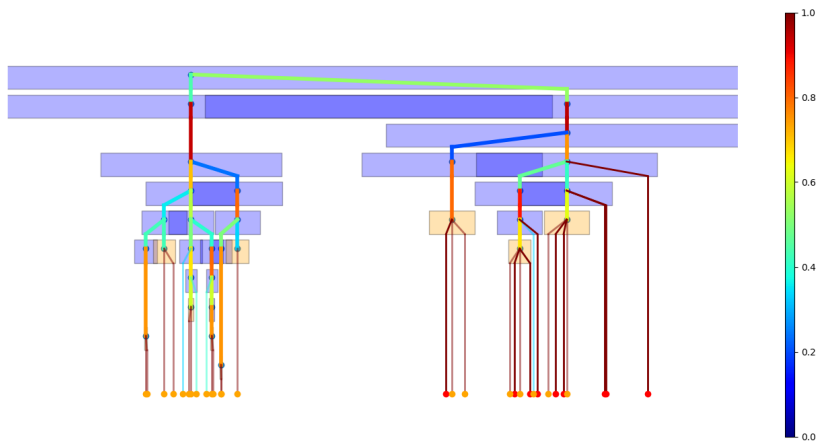
## A simple approximation of the true distribution

- ▶ Each node covers  $N$  elements of the tree.
- ▶ The node's children cover  $(N_1, N_2, \dots, N_k)$
- ▶ Therefore the probability of a point associated to the parent node, is associated to the  $i$ th child node is  $\frac{N_i}{N}$

# Approximating the Probability Distribution From a Covertree



# Oops, The Estimate was Wrong





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## Let's be Bayesian about this

- ▶ We know a lot about the root of the tree, lots of observations.
- ▶ We know little about the leaves of the tree, few observations.
- ▶ Therefore, model the distribution of distributions, using a Dirichlet distribution.

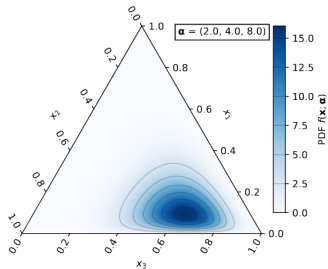
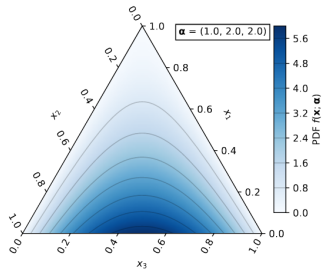
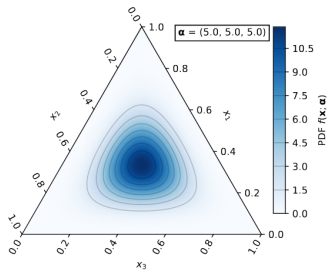
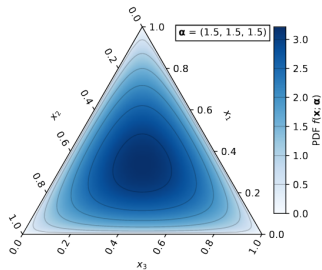
## A Node's Dirichlet Distribution

For node covering  $N_0$ , with children covering  $\alpha = (N_1, \dots, N_k)$ , we associate a Dirichlet Distribution  $\text{Dir}(\alpha)$ . The probability density function for this is:

$$f(x_1, \dots, x_k; N_1, \dots, N_k) = \frac{\prod_{i=1}^k \Gamma(N_i)}{\Gamma(N_0)} \prod x_i^{N_i-1}$$

Can also do this with all nodes for the "overall distribution"

# A Dirichlet Visualization <sup>1</sup>



<sup>1</sup>Source: Wikipedia

## Prior VS Posterior

The *prior* associated to a node is  $\text{Dir}((1, \dots, 1))$ . The training posterior is

$$P_A = \text{Dir}((N_1 + 1, \dots, N_k + 1)).$$

If there are  $O_i$  points in the test set whose paths pass through the  $i$ th child, then the test-posterior is:

$$Q_A = \text{Dir}((N_1 + O_1 + 1, \dots, N_k + O_k + 1)).$$

## Drift Metrics: Kullback–Leibler divergence <sup>2</sup>

$$\text{KL}(Q_A||P_A) = \log \Gamma(N_0) - \log \Gamma(N_0 + O_0) + \sum_{i=1}^k \{\Gamma(N_i + O_i) - \Gamma(N_i) + O_i(\psi(N_i) - \psi(N_0))\} \quad (1)$$

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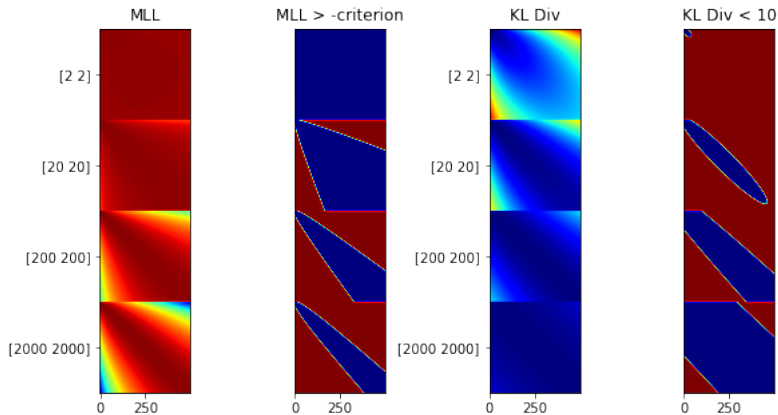
<sup>2</sup>Source: <https://bariskurt.com/kullback-leibler-divergence-between-two-dirichlet-and-beta-distributions/>

## Marginal Log Likelihood of Test, Given Observations

Model the distributions of multinomial distributions with  $O$  samples instead of categorical, then calculate the ln of the marginal distribution:

$$\text{MLL}(O|N) = \log \Gamma(N_0) + \log \Gamma(O_0 + 1) - \log \Gamma(N_0 + O_0) + \sum_{i=1}^k \{\Gamma(N_i + O_i) - \Gamma(N_i) - \Gamma(O_i + 1)\} \quad (2)$$

# Visualization Of KL Div VS MLL





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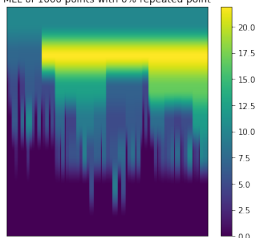
Results

## Let's build some intuition

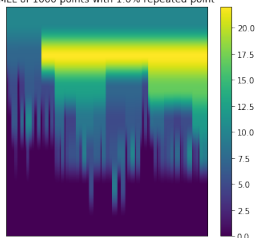
1. Our training set will be 10000 points from a 2D gaussian.
2. Our test sets will be 1000, and 10000 points sampled from the same gaussian.
3. We'll sample the attack point from the same gaussian.
4. We'll replace 0%, 1% and 10% of the test set with the attack point, these are the attack rates.

# Visualization Of Gaussian Toy

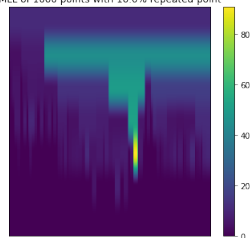
MLL of 1000 points with 0% repeated point



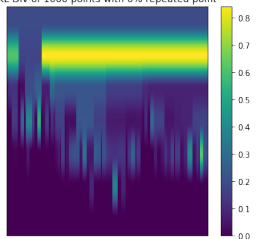
MLL of 1000 points with 1.0% repeated point



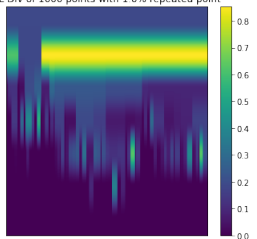
MLL of 1000 points with 10.0% repeated point



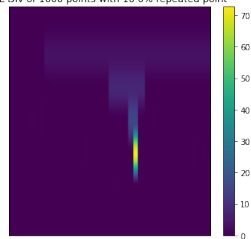
KL Div of 1000 points with 0% repeated point



KL Div of 1000 points with 1.0% repeated point

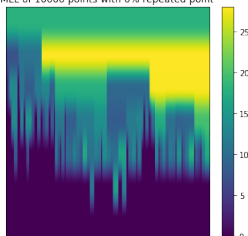


KL Div of 1000 points with 10.0% repeated point

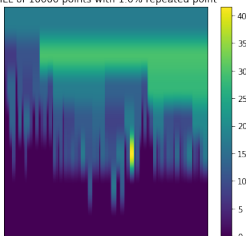


# Visualization Of Gaussian Toy

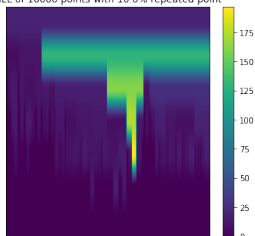
MLL of 10000 points with 0% repeated point



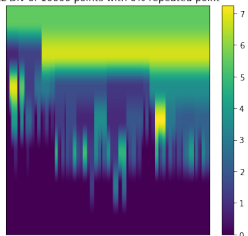
MLL of 10000 points with 1.0% repeated point



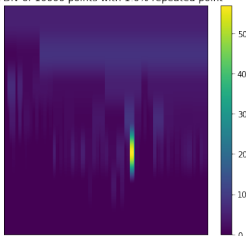
MLL of 10000 points with 10.0% repeated point



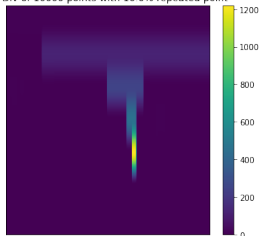
KL Div of 10000 points with 0% repeated point



KL Div of 10000 points with 1.0% repeated point



KL Div of 10000 points with 10.0% repeated point



## How to do Classification

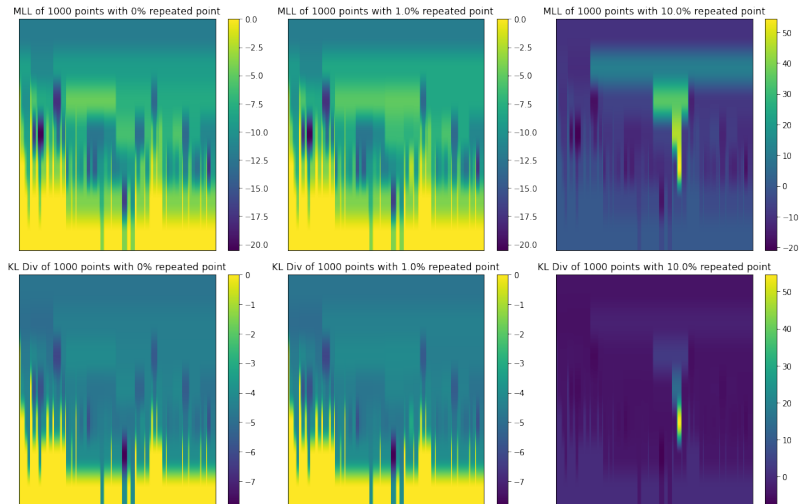
Take a baseline,  $B$ , run some sequences through the covertree's tracker and calculate the per-node maximum, and standard deviation.

$$\widehat{KL}_B(Q_a||P_a) = KL(Q_a||P_a) - \max_B KL(Q_a||P_a) - S_{KL}\sigma_{KL} - C_{KL}$$

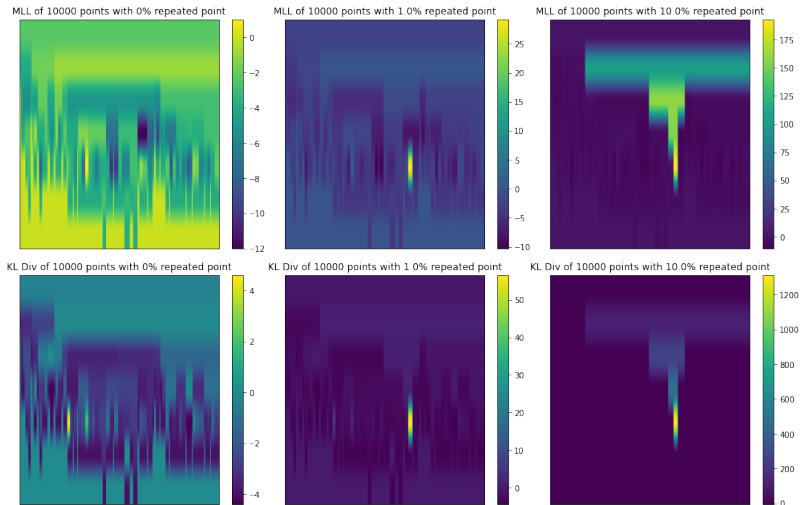
$$\widehat{MLL}_B(O||N) = MLL(O||N) - \max_{b \in B} MLL(O_b||N_b) - S_{MLL}\sigma_{MLL} - C_{MLL}$$



# Visualization Of Gaussian Toy



# Visualization Of Gaussian Toy



## Definition of Detection

A "detection" is performed in 2 passes, the first is the address of the node with the maximal positive  $\widehat{KL}_B(Q_a||P_a)$ .

If  $\widehat{KL}_B(Q_a||P_a)$  is everywhere non-positive, the address of the node with maximal positive  $\widehat{MLL}_B(O||N)$ .

If both terms are non-positive for all nodes, nothing is detected.



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## Overall KL Divergence of SOREL's test set

	Window size					
	1000		10000		100000	
Attack Rate	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$
0.0	0.0	1.0	0.0	1.0	0.0	1.0
0.0001	8e-5	1.0	3e-5	1.0	2e-5	0.999
0.001	0.0001	0.99	0.0003	1.0	0.0004	1.0
0.01	0.007	1.03	0.009	1.06	0.014	1.095
0.10	0.293	4.025	0.299	4.167	0.329	4.122
1.00	10.172	55.40	7.379	41.376	5.987	36.260

# Overall Marginal Log Likelihood of SOREL's test set

	Window size					
	1000		10000		100000	
Attack Rate	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$
0.0	0.0	1.0	0.0	1.0	0.0	1.0
0.0001	-0.0006	1.0	-0.0014	1.0	-0.0053	1.00
0.001	-0.004	0.99	-0.04	1.0	-0.16	1.00
0.01	-0.18	1.03	-0.92	1.08	-2.70	1.095
0.10	-3.78	1.66	-13.45	1.75	-32.26	1.38
1.00	-53.91	4.382	-160.87	2.78	-407.52	1.456

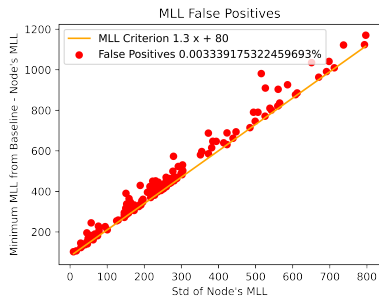
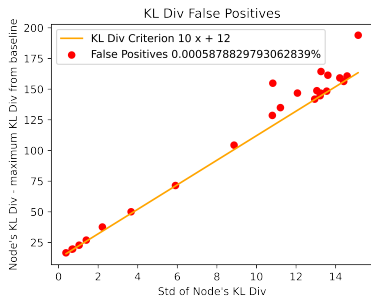
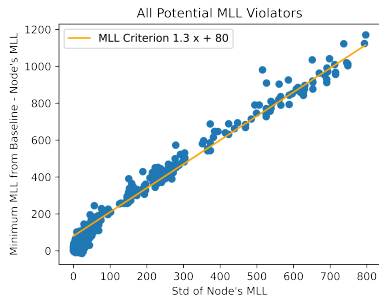
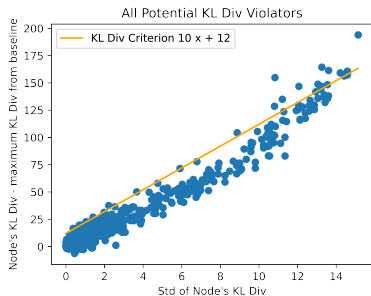
## SOREL Baseline Adjustment

Took a baseline, with a validation set. Did leave one out cross validation and adjusted the 4 hyperparameters until the following saw next to no FPS. There's an extra term  $\omega$  called the *margin of safety*. I used 1.5.

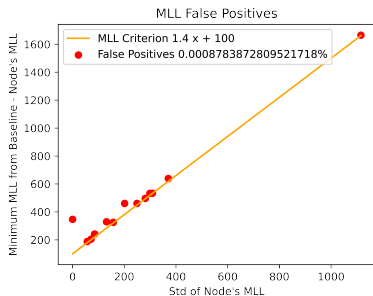
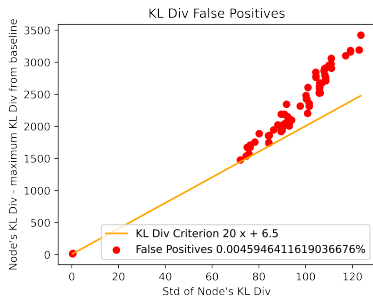
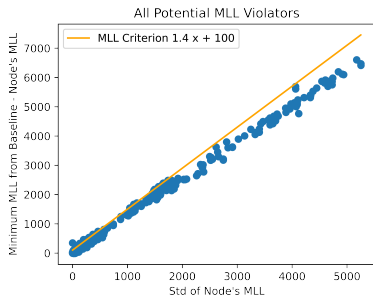
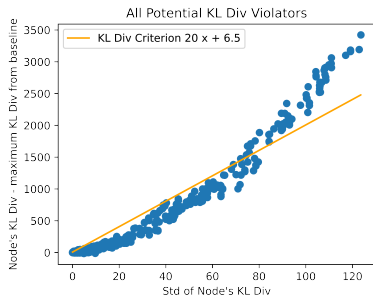
$$\widehat{\text{KL}}_B(Q_a||P_a) = \omega \text{KL}(Q_a||P_a) - \max_{B} \text{KL}(Q_a||P_a) - S_{\text{KL}}\sigma_{\text{KL}} - C_{\text{KL}}$$

$$\widehat{\text{MLL}}_B(O||N) = \omega \text{MLL}(O||N) - \max_{b \in B} \text{MLL}(O_b||N_b) - S_{\text{MLL}}\sigma_{\text{MLL}} - C_{\text{MLL}}$$

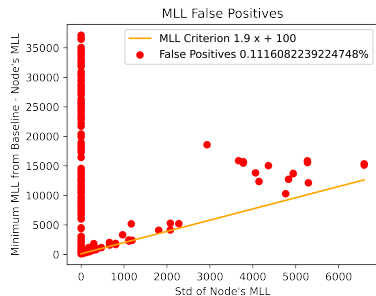
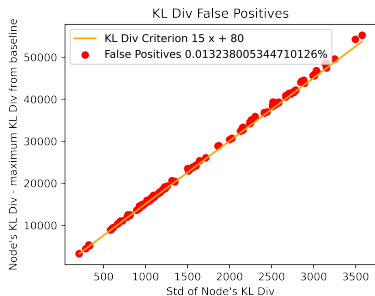
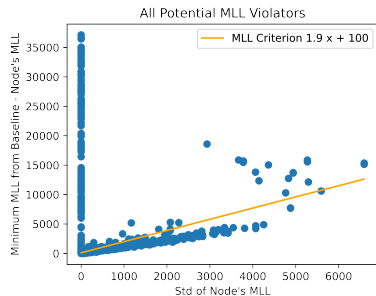
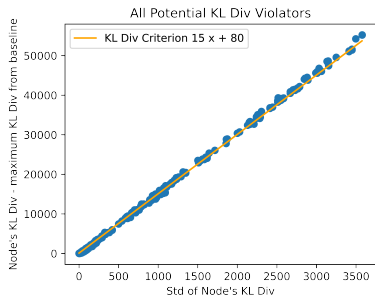
# Visualization Of SOREL Baseline Adjustment for 1000



# Visualization Of SOREL Baseline Adjustment for 10000



# Visualization Of SOREL Baseline Adjustment for 100000



# Safe Baseline Hyperparameter Results

With a safety margin of 2.

Window Size	$S_{KL}$	$C_{KL}$	$S_{ML}$	$C_{ML}$
1000	10	12	1.3	80
10000	20	6.5	1.4	100
100000	15	80	1.9	100



## Safe Test Set Results

		Attack Rates					
Window Size		0%	0.01%	0.1%	1%	10%	100%
1000	TPR	0	0	0	0.7	88	100
	FPR	0	0	0	0	0	0
	MDR	-	-	-	96	87	93
10000	TPR	0	0	0.7	63.7	99.95	100
	FPR	0	0	0	0	0	0
	MDR	-	-	96	93	93	91
100000	TPR	0	0.1	22.7	98.4	100	100
	FPR	0.4	0.3	0	0	0	0
	MDR	-	85	94	93	92	88

Mean Depth Rate - Detection depth of attack over the final depth. All values in percentages. Averaged over 1972 runs with 48 different trees.

# Not So Safe Baseline Hyperparameter Results

With a safety margin of 1.3.

Window Size	$S_{KL}$	$C_{KL}$	$S_{ML}$	$C_{ML}$
1000	8	7	1.3	20
10000	10	6.5	1.3	20
100000	10	40	1.7	50

## Not So Safe Test Set Attack Results for SOREL

		Attack Rates					
Window Size		0%	0.01%	0.1%	1%	10%	100%
1000	TPR	0	0	0	16.6	96	100
	FPR	0	0	0	0	0	0
	MDR	-	-	-	94	89	93
10000	TPR	0	0	5	81	99.95	100
	FPR	0	0	0	0	0	0
	MDR	-	-	94	91	93	91
100000	TPR	0	0.2	44.5	98.4	100	100
	FPR	0.1	0.9	0.6	0	0	0
	MDR	-	84	94	94	92	88

Mean Depth Rate - Detection depth of attack over the final depth. All values in percentages. Averaged over 1972 runs with 48 different trees.